Name of Course	: **BA. (Prog.) Mathematics**
Unique Paper Code	: 62354343_OC
Name of Paper	: Analytical Geometry and Applied Algebra
Semester	: III
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Describe sketch and label the focus, vertex and directrix of the parabola

$$4x^2 - 2y + 8x + 5 = 0.$$

Find the centre, vertices, foci and ends of minor axis of the ellipse

$$9x^2 + 4y^2 - 18x + 24y + 9 = 0.$$

Describe sketch and label the centre, vertices, foci and asymptotes of hyperbola

$$16x^2 - y^2 - 32x - 6y - 57 = 0.$$

2. Find equation of the parabola that has the focus $(-1, 4)$ and directrix $x = 5$.

Find equation of the hyperbola having foci at $(0, \pm 5)$ and asymptotes $y = \pm 2x$.

Rotate the coordinate axes to remove xy term and identify the curve

$$31x^2 + 10\sqrt{3}\,xy + 21y^2 - 144 = 0.$$

3. Find the vector projection of $\boldsymbol{v} = 6\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$ on $\boldsymbol{b} = \boldsymbol{i} - 2\boldsymbol{j} - 2\boldsymbol{k}$ and find the scalar component of $\boldsymbol{v}$ in the direction of $\boldsymbol{b}$.

Find the direction cosine of the vector $\boldsymbol{u} = 2\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$.

Determine whether the vectors $\boldsymbol{u} = \boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{v} = 3\boldsymbol{i} - 2\boldsymbol{k}$, and $\boldsymbol{w} = 5\boldsymbol{i} - 4\boldsymbol{j}$ lie in the same plane?

4. Find the distance between the skew lines:

$$L_1: x = 1 + 7t,\ y = 3 + t,\ z = 5 - 3t;$$

$$L_2: x = 4 - t,\quad y = 6,\quad z = 7 + 2t;$$

Determine whether the planes $3x - 4y + 5z = 0$ and $6x - 8y - 10z - 4 = 0$ are parallel or perpendicular to each other.

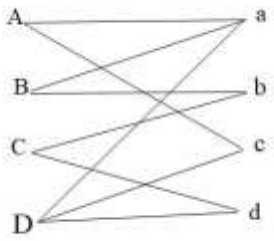
Find the angle between the planes

$$3x - 6y - 2z = 15 \text{ and } 2x + y - 2z = 5.$$

5. Find the parametric equations of the line through the point $(5, 0, -2)$ and is parallel to the planes $x - 4y + 2z = 0$ and $2x + 3y - z + 1 = 0$.

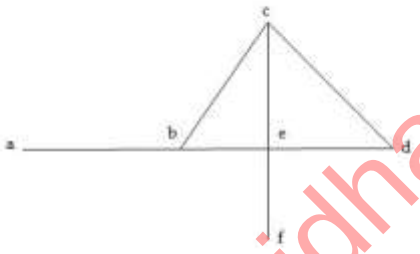
Define a Latin Square. Give an example of a Latin Square of order 4.

Find a matching or explain why none exists for the following graph.



6 Three pitchers of sizes 8 litre, 5 litre and 3 litre are given. If, initially, 8 litre pitcher is full and another two empty, find a minimal sequence of pouring so as to have exactly 1 litre of water in one pitcher.

Give a model of overlapping intervals for the following graph.



Find a maximum independent set in the following graph. Justify your answer.